COMPACTIFICATIONS OF SPLITTING MODELS OF PEL-TYPE SHIMURA VARIETIES — ERRATA

KAI-WEN LAN

(1) In Choices 2.2.9, "there exist some $r \in \mathbb{Z}$ and $\mathrm{j} \in \mathrm{J}$ such that $\Lambda = p^r \Lambda_{\mathrm{j}}$" should be "there exist some integers $(r_{[\tau]})_{[\tau] \in \Upsilon/\sim}$ and $\mathrm{j} \in \mathrm{J}$ such that $\Lambda_{[\tau]} = p^{r_{[\tau]}} \Lambda_{\mathrm{j},[\tau]}$, for all $[\tau] \in \Upsilon/\sim$, where $\Lambda_{[\tau]}$ and $\Lambda_{\mathrm{j},[\tau]}$ are the direct factors of Λ and Λ_{j}, respectively, as in (2.1.7)", and "p^{p_0}" should be "p^{r_0}".

(2) In the second paragraph of the proof of Prop. 2.2.11, the definitions of A_Λ and $f_{\Lambda,\Lambda'} : A_\Lambda \to A_{\Lambda'}$ (from the seventh to the ninth sentences) should become the following: "Hence, for any $\mathrm{j} \in \mathrm{J}$, with $\Lambda_{\mathrm{j}} = L_{\mathrm{j}} \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$ in $L \underset{\mathbb{Z}}{\otimes} \mathbb{Q}_p$, and for any $r \in \mathbb{Z}$, we can define $A_{p^r \Lambda_{\mathrm{j}}}$ to be the abelian scheme $\vec{A}_{\mathrm{j}}$ over $\vec{S}$. In general, for each $\Lambda \in \mathscr{L}$ such that $\Lambda_{[\tau]} = p^{r_{[\tau]}} \Lambda_{\mathrm{j},[\tau]}$ for some integers $(r_{[\tau]})_{[\tau] \in \Upsilon/\sim}$ and $\mathrm{j} \in \mathrm{J}$, for all $[\tau] \in \Upsilon/\sim$, as in Choices 2.2.9, there exists some $r \in \mathbb{Z}$ such that $r \geq r_{[\tau]}$, for all $[\tau] \in \Upsilon/\sim$, in which case we have a finite locally free subgroup scheme $\mathcal{K} := \prod\limits_{[\tau] \in \Upsilon/\sim} (\vec{A}_{\mathrm{j}}[p^{r - r_{[\tau]}}])_{[\tau]}$ of $\vec{A}_{\mathrm{j}}$ over $\vec{S}$, and we can define A_Λ to be the abelian scheme $\vec{A}_{\mathrm{j}}/\mathcal{K}$ over $\vec{S}$, with a canonically induced isogeny $f_{p^r \Lambda_{\mathrm{j}},\Lambda} : A_{p^r \Lambda_{\mathrm{j}}} \to A_\Lambda$. For any $\Lambda' \in \mathscr{L}$ such that $\Lambda \subset \Lambda'$ and $\Lambda'_{[\tau]} = p^{r'_{[\tau]}} \Lambda_{\mathrm{j}',[\tau]}$ for some integers $(r'_{[\tau]})_{[\tau] \in \Upsilon/\sim}$ and $\mathrm{j}' \in \mathrm{J}$, for all $[\tau] \in \Upsilon/\sim$, as in Choices 2.2.9, so that we have a similarly defined isogeny $f_{p^{r'} \Lambda_{\mathrm{j}'},\Lambda'} : A_{p^{r'} \Lambda_{\mathrm{j}'}} \to A_{\Lambda'}$, we define $f_{\Lambda,\Lambda'} : A_\Lambda \to A_{\Lambda'}$ to be the $\mathbb{Q}^\times$-isogeny given by the composition of $f_{p^{r'} \Lambda_{\mathrm{j}'},\Lambda'} \circ \vec{f}_{\mathrm{j},\mathrm{j}'} \circ f^{-1}_{p^r \Lambda_{\mathrm{j}},\Lambda}$ with multiplication by $p^{r - r'}$ on $A_{\Lambda'}$. At any geometric point $\bar{s} \to S$, the level structures $\alpha_{\mathcal{H}_{\mathrm{j}}}$ and $\alpha_{\mathcal{H}_{\mathrm{j}'}}$ compatibly induce isomorphisms matching the submodules $(L_{\mathrm{j}} \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^p) \times \Lambda$ and $(L_{\mathrm{j}'} \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^p) \times \Lambda'$ of $L \underset{\mathbb{Z}}{\otimes} \mathbb{A}^\infty \cong (L \underset{\mathbb{Z}}{\otimes} \mathbb{A}^{\infty,p}) \times (L \underset{\mathbb{Z}}{\otimes} \mathbb{Q}_p)$ with the submodules $\mathrm{T}\, A_{\Lambda,\bar{s}}$ and $\mathrm{T}\, A_{\Lambda',\bar{s}}$ of $\mathrm{V}\, A_{\bar{s}}$, respectively, so that the conditions in Definition 2.2.1 hold over the open dense subscheme S of $\vec{S}$, and therefore also over the whole $\vec{S}$."

(3) In the third paragraph of the proof of Prop. 2.2.11, the first two sentences should become the following: "For any $\mathrm{j}_0 \in \mathrm{J}$, since $\Lambda_{\mathrm{j}_0} \subset p^{r_0} \Lambda_0$ (see Choices 2.2.9), we have an isogeny $f_{p^{-r_0} \Lambda_{\mathrm{j}_0},\Lambda_0} : A_{p^{-r_0} \Lambda_{\mathrm{j}_0}} = \vec{A}_{\mathrm{j}_0} \to A_{\Lambda_0}$, as in the previous paragraph, and we can define the $\mathbb{Q}^\times$-polarization $\lambda_{\Lambda_0} : A_{\Lambda_0} \to A^\vee_{\Lambda_0}$ to be $(f^\vee_{p^{-r_0} \Lambda_{\mathrm{j}_0},\Lambda_0})^{-1} \circ \vec{\lambda}_{\mathrm{j}_0} \circ f^{-1}_{p^{-r_0} \Lambda_{\mathrm{j}_0},\Lambda_0}$. Since the level structure $\alpha_{\mathcal{H}_{\mathrm{j}_0}}$ matches the submodules $(L \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^p) \times \Lambda_0$ and $(L^\# \underset{\mathbb{Z}}{\otimes} \hat{\mathbb{Z}}^p) \times \Lambda_0^\#$ of $L \underset{\mathbb{Z}}{\otimes} \mathbb{A}^\infty \cong (L \underset{\mathbb{Z}}{\otimes} \mathbb{A}^{\infty,p}) \times (L \underset{\mathbb{Z}}{\otimes} \mathbb{Q}_p)$ with the submodules $\mathrm{T}\, A_{\Lambda_0,\bar{s}}$ and $\mathrm{T}\, A^\vee_{\Lambda_0,\bar{s}}$ of $\mathrm{V}\, A_{\bar{s}}$, respectively, for each geometric point $\bar{s} \to S$, and since $\Lambda_0 \subset \Lambda_0^\#$ (see

Published in Trans. Amer. Math. Soc. 370 (2018), no. 4, pp. 2463–2515, doi:10.1090/tran/7088.

Lemma 2.2.2), the $\mathbb{Q}^\times$-isogeny λ_{Λ_0} defined above is a $\mathbb{Z}_{(p)}^\times$-multiple of an isogeny over S, and hence is also a $\mathbb{Z}_{(p)}^\times$-multiple of an isogeny over $\vec{S}$, again by [12, Prop. 3.3.1.5] and the noetherian normality of $\vec{S}$."

(4) The proof of Lem. 3.1.1 should become the following: "By [12, Lem. 3.4.3.1 and Prop. 3.3.1.5], any $\mathbb{Z}_{(p)}^\times$-isogeny of abelian schemes over $\mathsf{M}_\mathcal{H}$ (uniquely) extends to a $\mathbb{Z}_{(p)}^\times$-isogeny of semi-abelian schemes over $\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$ as soon as the source extends. Hence, the assertion of the lemma does not depend on the choice of A_Λ in its $\mathbb{Z}_{(p)}^\times$-isogeny class. Therefore, as in the proof of Proposition 2.2.11, for each $\Lambda \in \mathscr{L}$ such that $\Lambda_{[\tau]} = p^{r_{[\tau]}}\Lambda_{\mathrm{j},[\tau]}$ for some integers $(r_{[\tau]})_{[\tau]\in\Upsilon/\sim}$ and $\mathrm{j} \in \mathrm{J}$, for all $[\tau] \in \Upsilon/\sim$, as in Choices 2.2.9, and for $r \in \mathbb{Z}$ such that $r \geq r_{[\tau]}$, for all $[\tau] \in \Upsilon/\sim$, we can take A_Λ to be $\vec{A}_\mathrm{j}/\mathcal{K}$, where $\mathcal{K} = \prod\limits_{[\tau]\in\Upsilon/\sim} (\vec{A}_\mathrm{j}[p^{r-r_{[\tau]}}])_{[\tau]}$. Since $\vec{A}_\mathrm{j}$ extends to a semi-abelian scheme $\vec{A}_\mathrm{j}^{\mathrm{ext}}$ with additional structures over $\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$ by [13, Thm. 11.2] and [15, Thm. 6.1], $\mathcal{K}$ also extends to the closed subgroup scheme $\mathcal{K}^{\mathrm{ext}} := \prod\limits_{[\tau]\in\Upsilon/\sim} (\vec{A}_\mathrm{j}^{\mathrm{ext}}[p^{r-r_{[\tau]}}])_{[\tau]}$ of $\vec{A}_\mathrm{j}^{\mathrm{ext}}$, which is quasi-finite and flat over $\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$. Thus, we can define A_Λ^{ext} to be $\vec{A}_\mathrm{j}^{\mathrm{ext}}/\mathcal{K}^{\mathrm{ext}}$, by [12, Lem. 3.4.3.1, Prop. 3.3.1.5, and the same local argument as in the proof of Thm. 3.4.3.2]."

(5) The proof of Prop. 3.1.2 should become: "In the proof of Lemma 3.1.1, the quotient $\vec{A}_\mathrm{j}^{\mathrm{ext}} \to A_\Lambda^{\mathrm{ext}} = \vec{A}_\mathrm{j}^{\mathrm{ext}}/\mathcal{K}^{\mathrm{ext}}$, where $\mathcal{K}^{\mathrm{ext}} = \prod\limits_{[\tau]\in\Upsilon/\sim} (\vec{A}_\mathrm{j}^{\mathrm{ext}}[p^{r-r_{[\tau]}}])_{[\tau]}$, induces morphisms $(\underline{\mathrm{Lie}}^\vee_{\vec{A}_\mathrm{j}^{\mathrm{ext},\vee}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}})_{[\tau]} \to \underline{\mathrm{Lie}}^\vee_{A_\Lambda^{\mathrm{ext},\vee}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}}$ and $\underline{\mathrm{Lie}}_{\vec{A}_\mathrm{j}^{\mathrm{ext}}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}} \to \underline{\mathrm{Lie}}_{A_\Lambda^{\mathrm{ext}}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}}$ that can be canonically identified with multiplication by $p^{r-r_{[\tau]}}$ on $(\underline{\mathrm{Lie}}^\vee_{\vec{A}_\mathrm{j}^{\mathrm{ext},\vee}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}})_{[\tau]}$ and $\underline{\mathrm{Lie}}_{\vec{A}_\mathrm{j}^{\mathrm{ext}}/\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}}$, respectively, for each $[\tau] \in \Upsilon/\sim$. Thus, by decomposing everything into factors indexed by $[\tau] \in \Upsilon/\sim$ as in Section 2.1, the proposition follows from [15, Prop. 7.15] (which was based on a reduction first to the case where Σ is induced by auxiliary choices as in [13, Sec. 7], and then to the good reduction case as in [11, Prop. 6.9])."

(6) In the proof of Lem. 3.2.22, the first sentence should become "First consider the special case where $\Lambda = p^r L_\mathrm{j} \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$, for some $r \in \mathbb{Z}$ and $\mathrm{j} \in \mathrm{J}$. By the construction of $A_\Lambda^{\mathrm{ext}} = \vec{A}_\mathrm{j}^{\mathrm{ext}}$ and $A_\Lambda^{\mathrm{ext},\vee} = \vec{A}_\mathrm{j}^{\mathrm{ext},\vee}$ over $\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$, which was based on [13, Lem. 11.1 and Thm. 11.2] and [15, Thm. 6.1] (or more precisely [15, Lem. 5.19 and Prop. 5.20]), their pullbacks to $(\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}})^\wedge_{\mathsf{Z}_{[(\Phi_\mathcal{H},\delta_\mathcal{H},\sigma)]}}$ are isomorphic to the pullbacks of the Mumford families ${}^\heartsuit\vec{G}_\mathrm{j}$ and ${}^\heartsuit\vec{G}_\mathrm{j}^\vee$ over $\vec{\mathfrak{X}}_{\Phi_\mathcal{H},\delta_\mathcal{H},\sigma}$ (see [12, Def. 6.2.5.28] and [13, (8.29)]), respectively." After the next sentence, "Then T_j, $T_\mathrm{j}^\vee$, ... we want" should be more precisely "In this case, T_j, $T_\mathrm{j}^\vee$ ... we want", and we need to insert a new sentence after this: "For general $\Lambda \in \mathscr{L}$, as in the proof of Lemma 3.1.1, we have an isogeny $\vec{A}_\mathrm{j}^{\mathrm{ext}} \to A_\Lambda^{\mathrm{ext}}$ of semi-abelian schemes over $\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}}$, for some $\mathrm{j} \in \mathrm{J}$, which induces isogenies of Raynaud extensions and of dual Raynaud extensions, by the constructions in [12, Sec. 3.3.3, 3.4.1, and 3.4.4], which give the desired T_Λ, $T_\Lambda^\vee$, (3.2.23), and (3.2.24) over $(\vec{\mathsf{M}}_{\mathcal{H},\Sigma}^{\mathrm{tor}})^\wedge_{\mathsf{Z}_{[(\Phi_\mathcal{H},\delta_\mathcal{H},\sigma)]}}$." Finally, in the last

sentence, "it suffices to note that for the polarization $\lambda_{\Lambda_0} : A_{\Lambda_0} \to A^\vee_{\Lambda_0}$ in Lemma 2.2.2, and for the $\mathrm{j}_0 \in \mathscr{L}$ such that $\Lambda_0 = p^{r_0} L_{\mathrm{j}_0} \underset{\mathbb{Z}}{\otimes} \mathbb{Z}_p$ for some $r_0 \in \mathbb{Z}$, we have a commutative diagram" should become "it suffices to note that, in the proof of Proposition 2.2.11, the polarization $\lambda_{\Lambda_0} : A_{\Lambda_0} \to A^\vee_{\Lambda_0}$ in Lemma 2.2.2 is defined to be $(f^\vee_{p^{-r_0}\Lambda_{\mathrm{j}_0},\Lambda_0})^{-1} \circ \vec{\lambda}_{\mathrm{j}_0} \circ f^{-1}_{p^{-r_0}\Lambda_{\mathrm{j}_0},\Lambda_0}$ over $\vec{\mathsf{M}}_{\mathcal{H}}$, for any $\mathrm{j}_0 \in \mathscr{L}$ (satisfying $\Lambda_{\mathrm{j}_0} \subset p^{r_0}\Lambda_0$ as in Choices 2.2.9), which (uniquely) extends to $(f^{\mathrm{ext},\vee}_{p^{-r_0}\Lambda_{\mathrm{j}_0},\Lambda_0})^{-1} \circ \vec{\lambda}^{\mathrm{ext}}_{\mathrm{j}_0} \circ (f^{\mathrm{ext}}_{p^{-r_0}\Lambda_{\mathrm{j}_0},\Lambda_0})^{-1}$ (with the superscript "ext" denoting the unique extensions of homomorphisms of semi-abelian schemes) over the noetherian normal scheme $\vec{\mathsf{M}}^{\mathrm{tor}}_{\mathcal{H},\Sigma}$ (by [12, Prop. 3.3.1.5]), and we have a commutative diagram".

(7) In the third paragraph of Thm. 3.4.1(4), the notation K for $\mathrm{Frac}(V)$ conflicts with the notation of K in earlier parts of the theorem. It should be changed to $\widetilde{K}$ (or some other symbol that has not been used).

(8) In Lem. 4.4.5, "a finite abelian group H_n of order prime to p" should be "a finite étale commutative group scheme H_n of order prime to p over $\vec{\mathsf{M}}^{\mathsf{Z}_{\mathcal{H}},\mathrm{spl}}_{\mathcal{H}}$".

University of Minnesota, Minneapolis, MN 55455, USA
Email address: kwlan@math.umn.edu